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Muon Number Violating Processes in Single Particle Extensions of the Standard Model

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Abstract

We study the one-loop induced muon number processes when the standard model is minimally extended to include a $SU(2)$ singlet of a charged scalar h^+ and a neutral fermion N . We find that $\mu \rightarrow e\gamma$ is more sensitive for the former model whereas $\mu - e$ conversion in nuclei for the latter. Effects of a scalar leptoquark $y^{1/3}$ and a heavy vector fermion E^- , which induce tree level rare muon decays, are also discussed.

Although the particle physics phenomenology can be well explained by the standard model (SM), most physicists believe that the standard model is not the final theory of the nature. There is an indication from recent experiments at LEP suggesting that the gauge coupling constants of the semi-simple gauge group of the standard model may meet at high energy [1]. This has given new impetus to the study of grand unified theories (GUTs) and their supersymmetric variants. Due to the dearth of experimental information on the one hand and a plethora of parameters in models beyond the SM such as GUT, we are unable to discriminate among models. In general these models contain many new particles some of which could be as light as a few hundreds GeV. If so, the low energy phenomenology could be affected due to the presence of these new particles.

In this paper, we adopt a bottom up approach in unravelling physics beyond the SM. We shall keep the SM gauge group and study the possible of extension of the SM by adding only one new particle to it at a time and study the low energy phenomenology of this new particle. By restricting ourselves to the SM gauge group, we are led to adding either a fermion or a scalar. Since we are adding only one new particle, it can be a SU(2) singlet if it is charged or a SU(2) \times U(1) singlet if it is neutral. With the addition of a new particle, we construct additional renormalizable interactions. We do not consider non-renormalizable interactions as they are suppressed by the high mass scale.

Let us first consider the addition of a new fermion. At first sight, it may appear that many possible fermions can be added. In fact, the choice is rather limited. For a SU(2) \times U(1) singlet, the choice is either a right-handed neutrino ν_R or a neutral Dirac (vectorlike) particle. For a charged SU(2) singlet, we are limited to having either a color singlet (E^-) or a color triplet ($U^{2/3}$ or $D^{-1/3}$). Due to the anomaly consideration, these charged fermions have to be vectorlike. We do not consider fermions with more exotic charges because more new scalars are required in order for them to interact with the standard particles through the Yukawa interactions. This would violate our philosophy of introducing only one particle.

Next we consider adding a scalar. The choice is either a singly charged (h^+), fractionally charged color triplets ($y^{1/3}$) or ($x^{1/3}$, $x^{2/3}$, or $x^{4/3}$) for a SU(2) singlet, where the particles

h^+ , $y^{1/3}$ and x 's are named as the dilepton, leptoquark and diquarks because they couple to lepton-lepton, lepton-quark and quark-quark pairs respectively. For the case of neutral scalar, we can add a $SU(2) \times U(1)$ singlet h^0 .

In general, a new particle can contribute to various low energy phenomenology. In particular, rare muon decays is very sensitive to such new physics because they are absolutely forbidden in the SM. In this paper we consider how a new particle from the above list can induce rare muon decays. Thus, the diquarks x 's, and vector quarks $U^{2/3}$ and $D^{-1/3}$ are not relevant here. Furthermore the h^0 , has only interactions with the SM Higgs and gives rise to possible CP violations in the scalar potential. A priori it has no effect on lepton number violating processes. This leaves a scalar dilepton h^+ and a Dirac neutrino N^1 which could induce rare muon decays at the one-loop level as our main focus here. The particles, E^- and $y^{1/3}$ which can induce rare decays at tree level would be considered next.

We write the relevant effective Lagrangian relevant as

$$\begin{aligned} \mathcal{L} = & \frac{g}{2s_W m_W^2} s_W^2 \lambda_1 A^\mu \left[F_1 \overline{e_L} (q^2 \gamma_\mu - \not{q} q_\mu) \mu_L + F_2 \overline{e_L} i \sigma_{\mu\nu} q^\nu m_\mu \mu_R \right] + \frac{g}{c_W} \lambda_2 P_Z Z^\mu \overline{e_L} \gamma_\mu \mu_L \\ & + \frac{g^2}{2m_W^2} \lambda_3 \left[B_e \overline{e_L} \gamma^\mu e_L + B_u \overline{u_L} \gamma^\mu u_L + B_d \overline{d_L} \gamma^\mu d_L \right] \overline{e_L} \gamma^\mu \mu_L, \\ & + g \lambda_4 H [C_1 \overline{e_L} \mu_R + C_2 \overline{e_R} \mu_L], \end{aligned} \quad (1)$$

where H is the neutral SM Higgs scalar. g and m_W are the gauge coupling constant of $SU(2)$ and the mass of the W gauge boson. $s_W = \sin \theta_W$ where θ_W is the Weinberg angle. In Eq. (1), $\lambda_{1,2,3,4}$ are model dependent dimensionless parameters which consist of products of coupling

¹ If we add only one ν_R , we can combine it with a linear combination of the usual left-handed neutrinos to form a massive Dirac neutrino and adjust the Yukawa couplings constants to give a very small mass for this massive state. If it has a Majorana mass, then see-saw mass and mixing relationships are obtained. For both cases, the effects for rare muon decays are negligible. When there are more than one ν_R 's, we can avoid the see-saw mass and mixing relationships leading to significant effects on the muon number violating process [20].

constants and mixings. Since we do not expect direct $\mu - e - \gamma$ coupling at tree level, F_1 and F_2 will be induced at one-loop level. On the other hand, tree level $\mu - e - Z$ coupling is allowed in some models and P_Z would then be unity; otherwise it will be induced at one-loop level or higher. Similarly, for leptoquark models, B_u is given as the mass-squared ratio of W to $y^{1/3}$ and $B_e = B_d = 0$; otherwise they will be given by one-loop box diagrams. For $\mu - e - H$ vertex, we anticipate that it would be helicity suppressed, namely, $C_1 \propto m_\mu/m_W$ and $C_2 \propto m_e/m_W$ in the models we are interested in. Also, the process $\mu \rightarrow 3e$ induced by H exchange is further suppressed by the small Yukawa coupling constant $gm_e/(2m_W)$. However, $\mu - e - H$ vertex will be important when there is the flavor changing interaction involving the right-handed muon, for instance, the supersymmetric model considered in Ref. [2].

In this paper, we consider the muon number violating processes, $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ and $\mu\text{Ti} \rightarrow e\text{Ti}$ which present branching ratios are measured to be 4.9×10^{-11} [3], 1.0×10^{-12} [4] and 4.6×10^{-12} [5], respectively. These are calculated to be [6–8]

$$B(\mu \rightarrow e\gamma) = \frac{24\pi}{\alpha} |s_W^2 \lambda_1 F_2|^2, \quad (2)$$

$$B(\mu \rightarrow 3e) = 2L^2 + R^2 - 4s_W^2 \lambda_1 F_2 (2L + R) + 4(s_W^2 \lambda_1 F_2)^2 \left(4 \ln \frac{m_\mu}{2m_e} - \frac{13}{6} \right), \quad (3)$$

$$B(\mu - e) = \frac{1}{\Gamma_0} \frac{\alpha^3 G_F^2 m_\mu^5}{\pi^2} \frac{Z_{eff}^4}{Z} |F(-m_\mu^2)|^2 |Q_W|^2, \quad (4)$$

with

$$L = s_W^2 \lambda_1 F_1 + (-1 + 2s_W^2) \lambda_2 P_Z - \lambda_3 B_e, \quad (5)$$

$$R = s_W^2 \lambda_1 F_1 + 2s_W^2 \lambda_2 P_Z, \quad (6)$$

$$Q_W = \left[\frac{2}{3} s_W^2 \lambda_1 (F_2 - F_1) + \left(\frac{1}{2} - \frac{4}{3} s_W^2 \right) \lambda_2 P_Z - \frac{1}{2} \lambda_3 B_u \right] (2Z + N) \\ + \left[-\frac{1}{3} s_W^2 \lambda_1 (F_2 - F_1) + \left(-\frac{1}{2} + \frac{2}{3} s_W^2 \right) \lambda_2 P_Z - \frac{1}{2} \lambda_3 B_d \right] (Z + 2N), \quad (7)$$

where G_F is the Fermi four-fermion coupling constant. $F(-m_\mu^2) = 0.54$ [9] and $Z_{eff} = 17.6$ [10] are the nuclear form factor and the effective atomic number for the nuclei ${}^{48}_{22}\text{Ti}$. $\Gamma_0 = (2.590 \pm 0.12) \times 10^6 \text{ sec}^{-1}$ [11] is the muon capture rate for Ti.

A model with a charged scalar dilepton h^+ . When a new charged scalar h^+ is introduced to the SM, new Yukawa interactions, which are given by

$$-\mathcal{L}_Y(\text{new}) = f_1(\nu_e\mu - e\nu_\mu)h^+ + f_2(\nu_e\tau - e\nu_\tau)h^+ + f_3(\nu_\mu\tau - \mu\nu_\tau)h^+ + H.c. , \quad (8)$$

are allowed. A new scalar potential depicting the interaction of h^+ with the SM Higgs doublet field Φ can be constructed and is given by

$$V = m^2|h^+|^2 + \lambda|h^+|^4 + a|h^+|^2\Phi^\dagger\Phi + V(\Phi) , \quad (9)$$

We can see from Eq. (8) that adding a h^+ will break the family lepton numbers, (L_e, L_μ and L_τ) but preserve the total lepton number, L . In addition, h^+ which is named as dilepton carries $L = -2$. This model is a simplified version of the Zee model [12] in which two Higgs doublets are introduced. The main motivation there is to generation neutrino masses.

Owing to Eq. (8), the e - μ - τ universality is broken if $f_1 \neq f_2 \neq f_3$. In particular, Fermi coupling constant G_F , which is extracted from the muon lifetime, will be modified to be

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2} \left[1 + \frac{f_1^2}{g^2 y} \right] + O\left(\frac{f_{2,3}^4}{g^4 y^2}\right) \quad (10)$$

due to the exchange of h^+ , where $y = m_h^2/m_W^2$. The best constraint on f_1 can be obtained from the nuclear beta decay and K_{e3} decay. Normalized to the muon decay, the CKM elements become $(|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2)[1 - 2f_1^2/g^2 y] = 0.9979 \pm 0.0021$ [15]. By the unitarity of the CKM matrix, we obtain $f_1^2/y \leq 2.1 \times 10^{-3}$ at 90% C.L.. In addition, the f_2 and f_3 terms will break the $e - \mu$ universality in the τ decays $\tau \rightarrow e\nu\nu$ and $\tau \rightarrow \mu\nu\nu$. Experimentally, the updated world average for these two decays modes are [16] $17.77 \pm 0.15\%$ and $17.48 \pm 0.18\%$ respectively. Combining the constraints on f_1 and f_2 obtained from $\mu - \tau$ universality violation in the decays $\mu \rightarrow e\nu\nu$ and $\tau \rightarrow e\nu\nu$ [17] with the updated world averages of the tau lepton mass and lifetime, $m_\tau = 1770.0 \pm 0.4$ MeV and $\tau_\tau = (295.9 \pm 3.3) \times 10^{-15}$ s respectively, we obtain that the ratios $f_{1,2}^2/y$ (f_3^2/y) are bounded to be on the order 10^{-3} (10^{-2}) or less.

Since the family lepton numbers are explicitly violated by h^+ , rare muon decays, such as $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ as well as $\mu^- - e^-$ conversion in nuclei, are allowed by one-loop quantum

corrections due to h^+ . Let us first consider the usual photon penguin diagrams of $\mu - e$ transition, see Fig. 1. Here we have $\lambda_1 = (f_2 f_3)/(8\pi^2)$ and the corresponding charge radius and magnetic moment terms, F_1 and F_2 , are given by

$$F_1 = -\frac{1}{18}\frac{1}{y}, F_2 = -\frac{1}{12}\frac{1}{y}. \quad (11)$$

respectively.

Since $m_\mu \ll m_Z$, it is a good approximation to neglect the external momenta for the processes we are considering. The Z penguin diagrams are then explicitly given as

$$P_Z(a) = \sin^2 \theta_W \left[\frac{1}{\epsilon} + \frac{1}{4} - \frac{1}{2} \ln m_h^2 \right], \quad (12)$$

$$P_Z(b) = \left[-\frac{1}{2\epsilon} - \frac{1}{8} + \frac{1}{4} \ln m_h^2 \right], \quad (13)$$

$$P_Z(c+d) = \left(-\frac{1}{2} + \sin^2 \theta_W \right) \left[-\frac{1}{\epsilon} - \frac{1}{4} + \frac{1}{2} \ln m_h^2 \right]. \quad (14)$$

Summing Eq. (12) to (14), we find that the effective $\mu - e - Z$ vertex vanishes, in the approximation that neglects the external momenta. The corrections due to external momenta for the processes such as $\mu \rightarrow 3e$ and $\mu - e$ conversion are suppressed by a small factor m_μ^2/m_Z^2 . Hence, Z penguin diagrams can be neglected in this model.

For the two h^+ exchange box diagrams, we obtain $\lambda_3 = \lambda_1(f_1^2 + f_2^2)/g^2$, $B_e = -1/(4y)$ and $B_u = B_d = 0$. However, the contributions are small unless the coupling f 's are on the order of g .

Let us now consider the process $\mu \rightarrow e\gamma$ which the experimental bound on the branching ratio is 4.9×10^{-11} [3]. This translates into a better constraint for $f_{2,3}$, namely

$$\frac{f_2 f_3}{y} < 2.8 \times 10^{-4}. \quad (15)$$

To study the relative importance of the different rare muon decays, we construct the branching ratios for $\mu \rightarrow 3e$ and $\mu - e$ conversion in Ti with respect to $\mu \rightarrow e\gamma$. Explicitly, we obtain

$$R_1 = \frac{B(\mu \rightarrow 3e)}{B(\mu \rightarrow e\gamma)} = \frac{\alpha}{24\pi} \frac{3F_1^2 - 12F_1F_2 + 4F_2^2(4\ln \frac{m_\mu}{2m_e} - \frac{13}{6})}{F_2^2} = 5.7 \times 10^{-3}, \quad (16)$$

$$R_2 = \frac{B(\mu \text{Ti} \rightarrow e\text{Ti})}{B(\mu \rightarrow e\gamma)} = 5.3 \times 10^{-6} \frac{|(F_2 - F_1)(Z + \frac{1}{3}N)|^2}{|F_2|^2} = 2.7 \times 10^{-4}. \quad (17)$$

Clearly, $\mu \rightarrow e\gamma$ is the best probe of the charged scalar singlet model.

A model with a neutral Dirac fermion N . In many GUT models, there exist heavy neutral fermions. For example, the E_6 GUT model which was first considered in Ref. [21] has fermion states for each generations placed in a $\underline{27}$ representation. Thus, there are additional 12 fermion states in addition to the 15 SM fermion states. The new particles, given in terms of left-handed chirality, are color singlet fermions ($E, E^c, \nu_E, N_E^c, N^c, n$) and color triplet fermions (D, D^c). The representation of the new particles under the standard $SU(2) \times U(1)$ depends on the E_6 symmetry breaking scheme. Motivated by this GUT, we first consider the neutral particle N .

N , which is a $SU(2) \times U(1)$ singlet neutral fermion, can be either a Dirac or Majorana particle. The phenomenology of having a Majorana particle has been extensively studied in Ref. [20]. Here, we consider N being a Dirac particle. Since it is a $SU(2) \times U(1)$ singlet particle, a gauge invariant mass term

$$-\mathcal{L}_M = M_N \bar{N}_L N_R + H.c. , \quad (18)$$

is allowed. The new Yukawa interactions are given as

$$-\mathcal{L}_Y(\text{new}) = \sum_{\alpha=e,\mu,\tau} f_\alpha (\bar{\nu}_{\alpha L} \quad \bar{\alpha}_L) N_R \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix} + H.c. , \quad (19)$$

where the charged lepton states are defined to be the mass eigenstates. Owing to the fact that the usual neutrinos couple to the massive neutrino shown in Eq. (19), the definition of the massless neutrinos is not arbitrary. The flavor states are related to the mass eigenstates by $(\nu_e, \nu_\mu, \nu_\tau, N)_L^T = \mathcal{O} (\nu_1, \nu_2, \nu_3, \nu_4)_L^T$ where \mathcal{O} is given by

$$\mathcal{O} = \begin{pmatrix} c_1 & s_1 c_2 & s_1 s_2 c_3 & s_1 s_2 s_3 \\ -s_1 & c_1 c_2 & c_1 s_2 c_3 & c_1 s_2 s_3 \\ 0 & -s_2 & c_2 c_3 & c_2 s_3 \\ 0 & 0 & -s_3 & c_3 \end{pmatrix} . \quad (20)$$

s_i are abbreviation of $\sin \theta_i$ which are given as $s_1 = f_e / \sqrt{f_e^2 + f_\mu^2}$, $s_2 = \sqrt{f_e^2 + f_\mu^2} / f$ and $s_3 = m_D / M_N$ where $f = \sqrt{f_e^2 + f_\mu^2 + f_\tau^2}$ and $m_D = f < \phi^0 >$. ν_1, ν_2 and ν_3 remain massless;

whereas ν_{4L} combines with N_R to form a Dirac neutrino with a mass equal to $\sqrt{m_D^2 + M_N^2}$. Therefore, the existence of the N will break the individual lepton flavor number conservation, leading to rare muon decays such as $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ and $\mu - e$ conversion in nuclei. However, the total lepton number remains conserved.

The presence of N would lead to the flavor changing neutrino- Z gauge coupling [18] which is given by

$$\begin{aligned} \mathcal{L}_{Z\nu\nu} = & \frac{g}{4c_W} Z_\mu \left[\overline{\nu}_{1L} \gamma^\mu \nu_{1L} + \overline{\nu}_{2L} \gamma^\mu \nu_{2L} + c_3^2 \overline{\nu}_{3L} \gamma^\mu \nu_{3L} \right. \\ & \left. + s_3^2 \overline{\nu}_{4L} \gamma^\mu \nu_{4L} + s_3 c_3 \overline{\nu}_{3L} \gamma^\mu \nu_{4L} + s_3 c_3 \overline{\nu}_{4L} \gamma^\mu \nu_{3L} \right] . \end{aligned} \quad (21)$$

Since we expect the new particle to come from the higher mass scale, it is reasonable to assume $m_4 > m_Z$. Thus, the invisible width of Z would be reduced due to the smaller coupling to ν_3 . The number of light neutrino species in the Z decay is given by

$$N_\nu = 2 + (1 - s_3^2)^2 . \quad (22)$$

N_ν is experimentally measured to be 2.980 ± 0.027 at the LEP [14], leading to

$$s_3^2 \leq 3.05 \times 10^{-2} (90\% \text{C.L.}) . \quad (23)$$

The neutrino mixings would also contribute to the $e - \mu - \tau$ universality violation [19,20], similar to our previous discussion. However, the constraint Eq. (23) is more appropriate for our discussions of rare muon decays.

Although the neutrino flavor is violated at tree level in the Z gauge interaction given by Eq. (21), rare muon decays are induced at one-loop level. From Eq. (20) and consideration of the γ penguin, we extract $\lambda_1 = g^2/(16\pi^2) \mathcal{O}_{\mu 4}^* \mathcal{O}_{e 4}$. The charge radius and the magnetic moment terms for the photon $\mu - e$ transition [22] are

$$F_1 = \frac{x_4(12 + x_4 - 7x_4^2)}{12(x_4 - 1)^3} + \frac{x_4^2(-12 + 10x_4 - x_4^2)}{6(x_4 - 1)^4} \ln x_4 , \quad (24)$$

$$F_2 = \frac{x_4(1 - 5x_4 - 2x_4^2)}{4(x_4 - 1)^3} + \frac{3x_4^3}{2(x_4 - 1)^4} \ln x_4 , \quad (25)$$

where $x_4 = m_4^2/m_W^2$. For the Z penguin diagram, care has to be taken due to the flavor changing coupling, and the final result can be simply written as

$$P_Z = \left[-\frac{5x_4}{2(x_4 - 1)} + \frac{3x_4^2 + 2x_4}{2(x_4 - 1)^2} \ln x_4 \right] + \frac{1}{2} s_3^2 x_4, \quad (26)$$

and $\lambda_2 = g^2/(32\pi^2)\mathcal{O}_{\mu 4}^*\mathcal{O}_{e4}$. The integrals B_e , B_u and B_d of the usual W -boson exchange box diagrams for $\mu \rightarrow 3e$ and $\mu - e$ conversion are given by

$$B_e = \left[\frac{x_4}{x_4 - 1} - \frac{x_4}{(x_4 - 1)^2} \ln x_4 \right] + |\mathcal{O}_{e4}|^2 \left[\frac{-4x_4 + 11x_4^2 - x_4^3}{4(x_4 - 1)^2} - \frac{3x_4^3}{2(x_4 - 1)^3} \ln x_4 \right] \quad (27)$$

$$B_u = \frac{4x_4}{x_4 - 1} - \frac{4x_4}{(x_4 - 1)^2} \ln x_4 \quad (28)$$

$$B_d = \frac{x_4}{x_4 - 1} - \frac{x_4}{(x_4 - 1)^2} \ln x_4 \quad (29)$$

and $\lambda_3 = \lambda_2$.

Let us first apply the above expressions to $\mu \rightarrow e\gamma$. Using the experimental bound given in Ref. [3], we obtain

$$|\lambda_1 F_2| \leq 3.0 \times 10^{-7}. \quad (30)$$

This translates into $\lambda_1 \leq 2.4 \times 10^{-6}$ (6.7×10^{-7}) for $m_4 = m_W$ ($10 m_W$).

Next we consider the process $\mu \rightarrow 3e$. From the expressions for the branching ratios given in Eqs. (2) and (3), the ratio $R_1 = B(\mu \rightarrow 3e)/B(\mu \rightarrow e\gamma)$ is proportional to a small factor $\alpha/(24\pi) = 1 \times 10^{-4}$. Naively, one would expect the branching ratio for $\mu \rightarrow 3e$ to be always very small. In fact, this is not the case because the process $\mu \rightarrow 3e$ receives large contributions from the Z penguin diagrams. When we include also the box diagram contributions, we obtain $R_1 \leq 0.03$ (0.10) [23] for $m_4 = m_W$ ($10 m_W$). Since the present experiment sensitivity for the experiment $\mu \rightarrow 3e$ is 50 times better than that of $\mu \rightarrow e\gamma$, the former experiment is better in probing the physics of a heavy Dirac neutral fermion N .

For the $\mu\text{Ti} \rightarrow e\text{Ti}$ experiment, the ratio of the branching ratio relative to that of $\mu \rightarrow e\gamma$ can be written as

$$R_2 = 5.1 \times 10^{-5} \frac{|Q_W|^2}{|s_W^2 \lambda_1 F_2|^2}. \quad (31)$$

Putting in limits for the various parameters, we obtain $R_2 \leq 12$ (6.3) [23] for $m_4 = m_W$ ($10 m_W$). Therefore, this experiment is far better than the other two.

Tree Level Rare Muon decays As we have discussed previously, a new heavy lepton E^- can exist in GUT models such as E_6 . Due to anomaly considerations, E has to be vectorlike under the SM gauge group. Thus the Yukawa interactions in the lepton sector and the gauge invariant mass term for E are given as

$$-\mathcal{L} = h_{ij}\overline{L^i}e_R^j\Phi + f_{i4}\overline{L^i}E_R\Phi + M_E\overline{E_L}E_R + H.c. , \quad (32)$$

where $i, j = 1, 2, 3$ are the family indices. L (e_R) is the usual $SU(2)$ doublet (singlet) leptons. When the neutral Higgs acquires vacuum expectation values $\langle \phi^0 \rangle$, the charged leptons get a mass matrix $M_l = (h + f)\langle \phi^0 \rangle + M_E \text{diag}(0, 0, 0, 1)$. We can diagonalize this mass matrix by a bi-unitary transformation $U_L^\dagger M_l U_R$. As shown in Eq. (32), the charged lepton masses come from two sources, namely $\langle \phi^0 \rangle$ and M_E . Thus, the tree-level lepton flavor changing Higgs vertex is induced. However, it is suppressed by m_μ/m_W .

On the other hand, flavor changing Z coupling is also induced by E because it has different gauge transformation in the left-handed sector. This leads to large tree level contributions to $\mu \rightarrow 3e$ and $\mu - e$ conversion. At the tree level we have $\lambda_1 = \lambda_3 = 0$, and $\lambda_2 = (1/2)U_{Le}^*U_{L\mu}$ with $P_Z = 1$. In this case, both processes $\mu\text{Ti} \rightarrow e\text{Ti}$ and $\mu \rightarrow 3e$ proceed through the lepton flavor changing Z coupling at the tree level. Thus, the ratio $B(\mu\text{Ti} \rightarrow e\text{Ti})/B(\mu \rightarrow 3e)$ is approximately equal to 10. This implies that the best probe for the tree level flavor changing Z coupling induced by E^- is the $\mu - e$ conversion experiment and we arrive at the bound $U_{Le}^*U_{L\mu} < 1.5 \times 10^{-6}$.

The second example for this class of model is the existence of a leptoquark $y^{1/3}$ which induces a new Yukawa interaction $f_{ij}L^iQ^jy^{1/3}+H.c.$, where L and Q are the lepton and quark doublets where i and j are the generation indices. In this case, $\lambda_1 = \lambda_2 = B_e = B_d = 0$, and $\lambda_3 = f_{\mu u}f_{eu}^*/g^2$ and $B_u = -(m_W/m)^2$, where m is the mass for the $y^{1/3}$. Therefore, in this model only $\mu - e$ conversion is allowed at the tree level by the exchange of $y^{1/3}$, leading to a stringent constraint $f_{\mu u}f_{eu}^*(m_W/m)^2 < 1.1 \times 10^{-7}$.

In conclusion, we have studied the effects on the muon number violating decays induced by adding one new particle to the SM. When the rare muon decays such as $\mu \rightarrow e\gamma$, $\mu \rightarrow 3e$ and $\mu\text{Ti} \rightarrow e\text{Ti}$ are induced by new physics at the one-loop level, one would expect that the first process to be dominant because it is kinematically more favorable. However, this is not necessary always true. We find that for the case of a charged scalar h^+ , $\mu \rightarrow e\gamma$ is the most sensitive experiment to probe a charged scalar singlet h^+ . On the other hand, $\mu - e$ conversion in nuclei is the best probe for a heavy Dirac neutrino N because of the large contribution coming from the Z penguin diagrams. Tree level effects are also possible by adding an charged vectorlike lepton E^- or a leptoquark scalar $y^{1/3}$. Again, the most sensitive experiment is $\mu - e$ conversion in nuclei. In table I, we present the constraints for λ 's obtained from these three muon number violating processes with the masses for the new particles taken to be m_W and $10 m_W$. For completeness, we include the result given in Ref. [20] for the case of adding right-handed Majorana neutrinos.

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TABLES

TABLE I. The constraints for λ 's obtained from the processes $\mu \rightarrow e \gamma$, $\mu \rightarrow 3e$, and $\mu \text{ Ti} \rightarrow e \text{ Ti}$, where the masses for the new particles are taken to be from m_W (10 m_W).

Model	$B(\mu \rightarrow e\gamma)$	$B(\mu \rightarrow 3e)$	$B(\mu\text{Ti} \rightarrow e\text{Ti})$
h^+	$\lambda_1 \lesssim 3.6 \times 10^{-6}(3.6 \times 10^{-4})$	$\lambda_1 \lesssim 6.7 \times 10^{-6}(6.7 \times 10^{-4})$	$\lambda_1 \lesssim 6.8 \times 10^{-5}(6.8 \times 10^{-3})$
N	$\lambda_1 \lesssim 2.4 \times 10^{-6}(6.7 \times 10^{-7})$	$\lambda_1 \lesssim 1.8 \times 10^{-6}(2.8 \times 10^{-7})$	$\lambda_1 \lesssim 2.1 \times 10^{-7}(8.0 \times 10^{-8})$
E^-	n/a	$\lambda_2 \lesssim 1.1 \times 10^{-6}$	$\lambda_2 \lesssim 7.7 \times 10^{-7}$
$y^{1/3}$	n/a	n/a	$\lambda_3 \lesssim 2.7 \times 10^{-7}(2.7 \times 10^{-5})$
ν_R ^a	$\lambda_1 \lesssim 2.4 \times 10^{-6}(6.7 \times 10^{-7})$	$\lambda_1 \lesssim 1.9 \times 10^{-6}(1.5 \times 10^{-7})$	$\lambda_1 \lesssim 2.1 \times 10^{-7}(6.9 \times 10^{-8})$

^a we include the result for the Majorana neutrino model studied in Ref. [20]

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FIGURES

FIG. 1. γ and Z penguin diagrams for μ - e transition for a scalar h^+ model

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